Agency Contracts in a Matching Market

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Structure of the talk

1. Introduction
   a) Moral hazard contracts
   b) Matching markets

2. Principal-agent markets (three examples)
   a) Relationship between risk and incentives (firms and managers)
   b) The nature of the matching under moral hazard (landlords and tenants)
   c) Long term vs short term contracts (firms and workers)

Moral Hazard

(Textbooks, e.g., Macho-Stadler and Pérez-Castrillo, 1997)

- Bilateral relationship between two (predetermined) players: P-A.
- A principal hires an agent to perform a task, $e \in E$, in exchange for a wage, w.
- The final outcome x depends on e and some random variable for which both participants have the same prior distribution.
- Principal risk-neutral with utility function $B(w, e) = x - w$
- Agent (possibly) risk-averse has an additively separable utility function $U(w, e) = u(w) - v(e)$.
- Risk aversion or limited liability constraints play an important role.
- The agent's reservation utility is exogenous: $\underline{U}$.

- Under moral hazard, to find the optimal contract, the principal solves:

$$\underset{\left\{(w(x))_{x\in X},e\right\}}{Max} \left\{E\left(x - w\left(x\right) \mid e\right)\right\}$$

$$\text{s.t. } E\left(u\left(w\left(x\right)\right) \mid e\right) - v(e) \geq \underline{U}$$

(Sym Information)

$$e \in \arg\max_{\widehat{e}\in E} \left\{E\left(u\left(w\left(x\right)\right) \mid \widehat{e}\right) - v(\widehat{e})\right\}$$

- From the Lagrangean

$$\frac{1}{u'(w^*(x))} = \lambda + \mu \frac{p_e(x \mid e)}{p(x \mid e)}$$

(Moral hazard)

- MH may give rise to several distortions in the optimal contract because it forces the principal to trade-off incentives for the effort of the agent and other objectives (most often efficiency – full insurance).

On the isolate P-A relationship

- Comparative static exercises allow to see how the parameters affect the optimal contract (the agent's degree of risk aversion, the cost of his effort or his productivity level; the profitability of the principal project, her productivity, ...)
- Comparative static on the agent's reservation utility allow to discuss that happens when the agent has more bargaining power.

- From the point of view of Moral Hazard, considering the market allows to endogenize the agent's "reservation utility" and the attributes of the matched pairs.

Matching market

(Textbook Roth and Sotomayor, 1990)

- In the P-A model P has all the bargaining power and A accepts a contract if he gets at least his (exogenously given) $\underline{U}$.

- When there is a market, Ps and As can look for alternative partners. In this market, in addition to optimal contracts, the identity of the pairs that meet (sign contracts) is endogenously determined. In particular:
  - the alternative relationships that could be formed in the market determine the endogenous level of payoffs that each principal and agent obtain and some properties of the contract.
  - In this setup the "reservation utility" in the market of an agent is endogenous, as well as the "attributes" of the matched partners.

From the point of view of matching

This is an extension of the Assignment Game - Shapley and Shubik's (1972)

- Two-sided one-to-one matching markets with
  - a finite set of heterogeneous buyers (principals)
  - a finite set of heterogeneous sellers (agents)
  - each seller (agent) is willing to sell at most one object (work at most for one principal)
  - each buyer (principal) wants to buy at most one object (hire at most one agent)
  - money is exchanged (an incentive contract is signed)

When utility is transferable

- Equilibrium outcomes satisfy two useful properties:
  - equilibrium contracts are always Pareto optimum (isolated relationships).
  - any equilibrium matching is efficient: it maximizes Total Surplus (the sum of all the profits and utilities in the market cannot be increased by reassigning principals and agents).
- As for which types matched in equilibrium
  - with no frictions: complementarities/supermodularity of the joint outcome => *Positive Assortative Matching* PAM
  - with frictions (market imperfections) it is more complicated (Legros and Newman, 2002)

When utility is not transferable

- Equilibrium outcomes still satisfy the property that the equilibrium contracts are always Pareto optimum (what we have learned from the analysis of isolated relationships).
- But this does not imply that total surplus is maximized.

- Kaneko (1982) proves existence of equilibria.
- Legros and Newman (2007) propose sufficient conditions for the matching outcome of a two-sided one-to-one game to be PAM or NAM when utility is not perfectly transferable. They show that in addition to the *complementarity in partners' types*, one has to consider the *complementarity between a player's types and her partner's payoff*.

We are interested in understanding:

A) How moral hazard affects which partnerships are formed

- Understand whether at equilibrium there is
  - *Positive Assortative Matching* (PAM).
  - *Negative Assortative Matching* (NAM).

We are interested in understanding:

B) How selection effects affect the contractual choices

- The choice of the contractual form is influenced by
  - The shape of the incentive considerations within a partnership (optimal contract in a given P-A partnership)
  - The allocation of P-A partnerships in a market (given the types/attributes/characteristics of the participants in both sides of the market)

Incentives vs risk sharing

Risk and incentives (CARA)

(Holmstrom and Milgrom, 1987)

This game is TU

- P is risk neutral
- $U(w, e) = -\exp[-r\,(w - v(e))]$
- $v(e) = (1/2)\, v\, e^2$
- $x = e + \varepsilon$ $\varepsilon \sim N(0, \sigma^2)$ Linear contract $w(x) = F + s\, x$
- $CE(w, e) = F + s\, e - (1/2)\, v\, e^2 - (r/2)\, s^2\sigma^2$

P/A

- Under symmetric information, the principal will fully ensure the agent, $e^{FB} = 1/v$.

Market

- There exists a set of principals who are heterogeneous in the risk: each principal is associated with the variance of her project $\sigma^2 \in [\sigma_L{}^2, \sigma_H{}^2]$
- There is a set of agents heterogeneous in their risk aversion attitude: each agent is identified by his degree of risk aversion $r \in [r_L, r_H]$
- Both populations have the same mass.
- Under Sym Info $$\frac{\partial^2 B_{ij}^{FB}}{\partial \sigma_i^2 \partial r_j} = 0$$
- As a consequence, *in a market* where principals are heterogeneous in the risk of their projects and the agents are heterogeneous in their risk aversion, under sym info any matching is an equilibrium matching (and for any distribution of types).

Under moral hazard

- $e^* = s / v$ (ICC)

- Incentives

Incentives decreasing in degree of risk aversion, cost of the effort and variability of the outcome

$$s^* = \frac{1}{1 + rv\sigma^2}$$

- A robust prediction of P-A literature is the negative relationship between risk and performance pay (when the agent has CARA risk preferences, the variable part of the contract s^* is decreasing in the risk of the relationship, σ^2)

Risk and incentives (in a market)

(Serfes, 2005)

Q: Does this conclusion still hold when Ps and As interact in a market?

The Sufficient Condition for PAM (NAM) is:

$\partial^2 \pi / \partial r \, \partial \sigma^2 \geq (\leq)\ 0 \quad \leftrightarrow \quad v\, r\, \sigma^2 - 1 \geq (\leq)\ 0$

Then, if $r_L\ \sigma_L^2 \geq 1 / v$ then PAM

If risk and/or degree of risk aversion are always large then low-risk averse As are matched with low-risk Ps.

And, if $r_H\ \sigma_H^2 \leq 1 / v$ then NAM

If risk and/or degree of risk aversion are always small then low-risk averse As are matched with high-risk Ps.

In a market, the relationship between risk σ^2 and bonus s depends on a:

- *Direct effect* of σ^2 on s (as in standard P-A model): always *negative*.

- *Indirect effect* of σ^2 on s through the assignment that may be *negative* (if PAM, because a high σ^2 is matched with a high r, which leads to a low s) or *positive* (if NAM).

- Then, if PAM the relationship between risk and performance pay is negative, but if NAM it can be positive or have any other shape (e.g., a U shape).

(Some intuitions for three-sided markets)

Incentives to the agent vs incentives to the principal

Double-sided Moral Hazard

(Eswaran and Kotwal, 1985)

- Both P and A are risk neutral, A chooses *e* at a cost *v(e)* and, simultaneously, P decides on her effort $a \in A$, at a cost of $c(a)$.

  Take $v(e) = (1/2)\, e^2$ and $c(a) = (1/2)\, a^2$

- The outcome depends on both *e* and *a* according to $x = h(e,a) + \varepsilon$

(one cannot do the effort of the other)

Take $h(e, a) = \alpha\, \theta_A\, \theta_P + \theta_A\, e + \theta_P\, a$ $(\theta_I \geq 1,\ I = A, P)$

There are complementarities $\alpha \geq 0$

- Contracts are linear:

This game is TU

$w(x) = F + s\, x$ $s \in [0,1]$

(s=0, F>0 wage // s=1, F<0 rent // $s \in (0,1)$ sharecropping)

- The program includes a new constraint: the principal's ICC.

Consequence: P looses her role as residual claimant.

Double Moral Hazard

- Then, under MH, the optimal payment scheme is $s^* = \dfrac{\theta_A^2}{\theta_A^2 + \theta_P^2}$

and

$$e^* = \theta_A s^* = \frac{\theta_A^3}{\theta_A^2 + \theta_P^2} \text{ and } a^* = \theta_P (1 - s^*) = \frac{\theta_P^3}{\theta_A^2 + \theta_P^2}.$$

In the FB: $e^{FB} = \theta_A$ and $a^{FB} = \theta_P$, then:

$$e^{FB} - e^* > a^{FB} - a^* \iff \theta_A < \theta_P.$$

Reversal in the nature of the marching

(Ghatak and Karaivanov, 2014)

- Ps and As are risk neutral. Populations (finite) with characteristic θ_P and θ_A
- Output $x = \alpha\, \theta_A\, \theta_P + \theta_A\, e + \theta_P\, a$ $(\alpha > 0)$
- <u>If efforts are contractible</u>, $e^{FB} = \theta_A$ and $a^{FB} = \theta_P$ and the joint surplus in a relationship is

$$S^{FB}(\theta_A, \theta_P) = \alpha\, \theta_A\, \theta_P + (1/2)\,(\theta_A^{\,2} + \theta_P^{\,2})$$

$S^{FB}(\theta_A, \theta_P)$ is increasing in θ_A and θ_P and the cross-partial derivative is positive $\rightarrow$ PAM: Ps with a high θ_P end up with agents with a high θ_A, and vice-versa (for any distribution of types).

- Under MH, total surplus is

$$S(\theta_A, \theta_P) = \alpha\theta_A\theta_B + \frac{1}{2}\left(\theta_A^2 + \theta_P^2\right) - \frac{1}{2}\frac{\theta_A^2\theta_P^2}{\left(\theta_A^2 + \theta_P^2\right)}$$

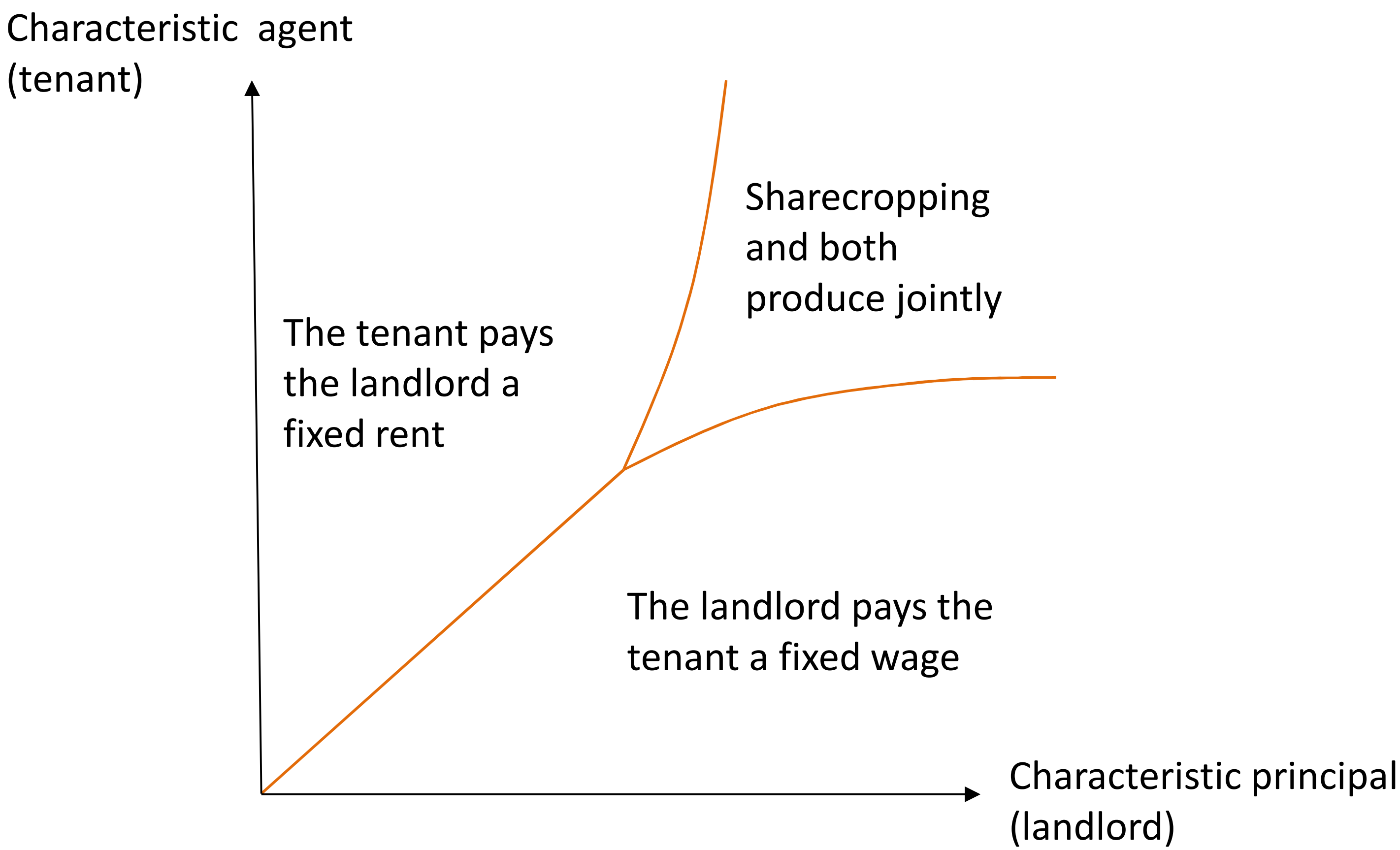
The cross-partial derivative is negative for (positive but) low values of α →

NAM: Ps with a high θ_P end up with agents with a low θ_A, and vice-versa

INTUITION

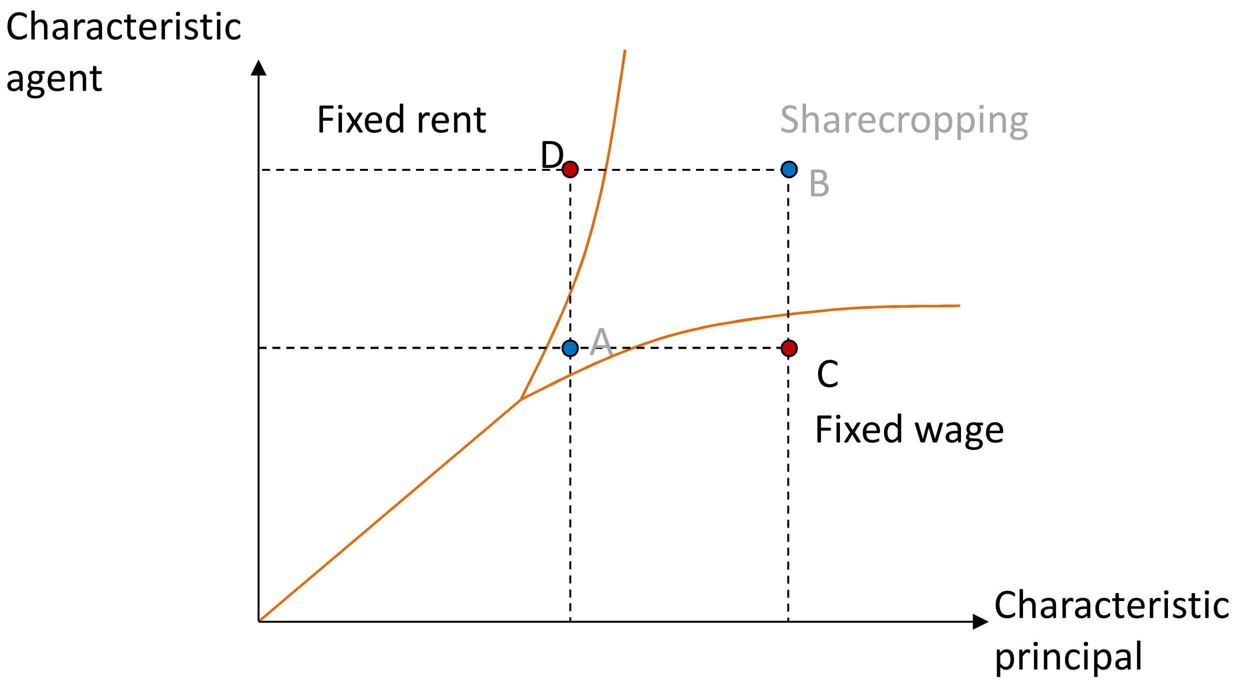
- The modularity of the joint surplus under MH depends on both the complementarity of the characteristics in the production function and on the endogenous efforts, which depend on the optimal sharing rule.
- Incentives depend on the relative characteristics. Better incentives are provided to θ_P when θ_A is low rather than when it is high.
- The positive effect of an increase in θ_P on *a* is lower the higher θ_A is. This effect induces a *substitutability* between the types that more than compensates the *complementarity* in the production function when α is low.

EXOGENEOUS P-A PAIR



- Taking pairs A and B in isolation, or if A and B are the exogenously matched partners: sharecropping in both cases

- If there are two participants in each side of the market, the match is endogenous and is NAM, then total surplus for C and D is higher than for A and B (hence sharecropping is dominated and it will not happen at equilibrium)



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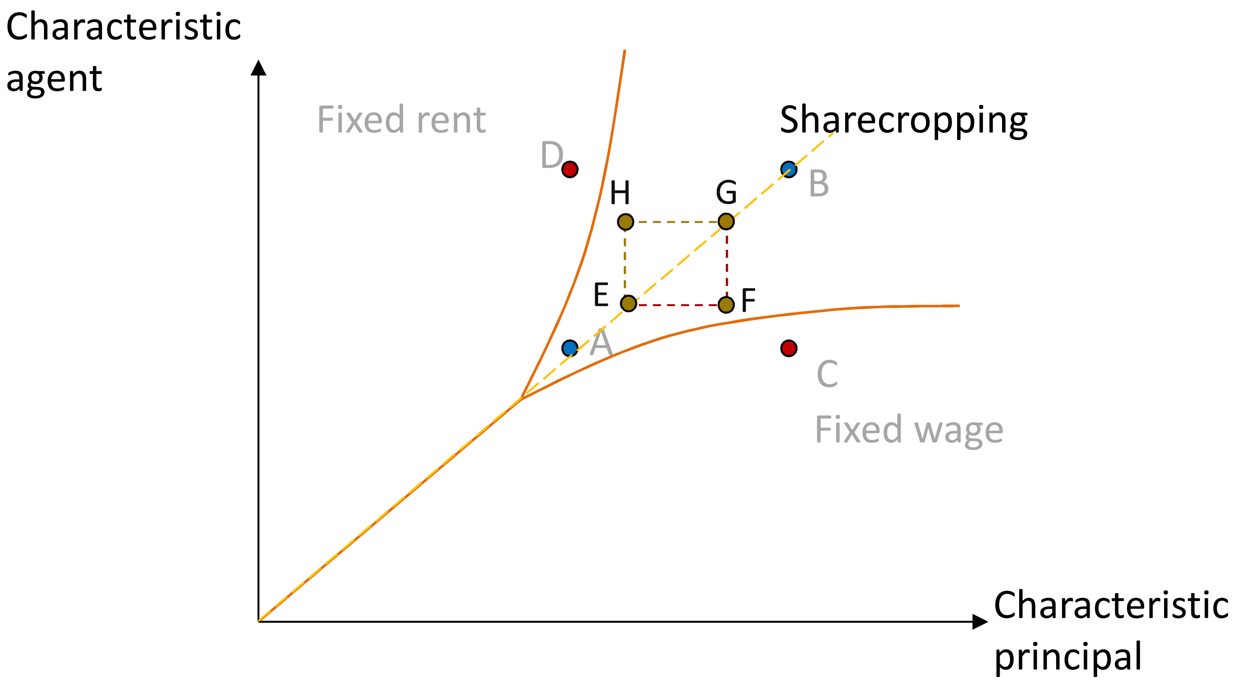


Long vs short term contracts

Repeated Moral Hazard

(Macho-Stadler, Pérez-Castrillo, and Porteiro, 2014)

- Overlapping generation model: at each period t (infinite-lived) firms contract with workers (who live for two periods) to develop projects.
- At $t+1$ the market is composed by the set of firms, the workers that enter the market at $t+1$ and the old workers that entered the market at period t.
- Each t, a firm is endowed with a project. The revenue from the project is $F+R$ if success, and F if failure. Firms differ w.r.t. profitability of projects R, distributed according to $G(R)$.
- To develop its project, each period, a firm needs a non-specialized worker (junior) and a specialized worker (senior). All workers are identical when junior (they are indistinguishable in ability). The seniors' ability are unknown to all ex-ante, it is revealed to all during the training period.

- Senior agents run the project; their effort is not verifiable. Probability of success is $p\,e$: where e is the senior's effort and p his ability. This ability p can take two values: p_L and p_H, with $p_L < p_H$.
- Only after being employed as a junior an agent's ability is public knowledge. There is a proportion q of p_H agents in the population.
- All participants are risk neutral and can commit to LT contracts.
- Workers, at any age, enjoy limited liability over income, $\underline{w} = 0$.
- Agents have an outside (this market) opportunity every period is equal to U^o (e.g., unemployed)

- A senior agent enters a relationship if his expected utility is at least equal to U^o. A junior agent accepts a contract if his expected intertemporal utility is at least $U^o + \delta U^o$.

- Junior agent's are hired under a fixed salary w_J is optimal.

- Seniors receive a contingent payment scheme, $C_S = (w_S, \Delta_S)$, $S = L, H$, a base payment and a bonus if R is obtained. It may depend on the project of the firm and the ability of the senior agent. The bonus will determine the senior's effort.

- Firms and workers can sign either short-term (ST) or long-term (LT) contracts.

- A ST contract with a junior agent is a salary w_J .

  A ST contract with a senior agent is an incentive scheme (w_S, Δ_S), for $S = H$ or L.

- A LT contract with a junior agent at period t specifies the salary that the worker will receive in period t and the incentive scheme to be used in period $t + 1$: $(w_J, w_H, \Delta_H, w_L, \Delta_L)$.

  A LT contract implies a commitment by the firm to keep the worker when senior, and a commitment by the agent to work for the same firm at period $t + 1$ independently of his ability.

- An *Equilibrium* is an outcome where:
  - all active workers obtain, at least, their outside utility (i.e., junior workers achieve an expected total utility of at least $(1+\delta)\,U^o$ and senior workers obtain at least U^o if they sign a contract at this age;
  - no firm would obtain higher expected intertemporal profits by changing the set of proposed contracts by another set of contracts that guarantee to each worker at least the same level of expected utility that he obtains under the current outcome.

- We concentrate the analysis on *stationary equilibria*, that is, on equilibria where firms follow the same strategy every period.

- At equilibrium, a firm may
  - offer LT contracts to junior workers (so that it keep them when senior whatever their type), or
  - offer ST contracts to junior workers and to senior workers of high, or low, ability.

- Therefore, at equilibrium the set of firms $[\underline{R}, \overline{R}]$ can be partitioned at most in three subsets:
  - the set R^{LT} of firms that offer LT contracts,
  - the set R^{H} that offer ST contracts and hire high-ability seniors and
  - the set R^{L} that offer ST contracts and hire low-ability seniors.

For one isolated firm, ST contracts are (at least weakly) dominated by LT contracts

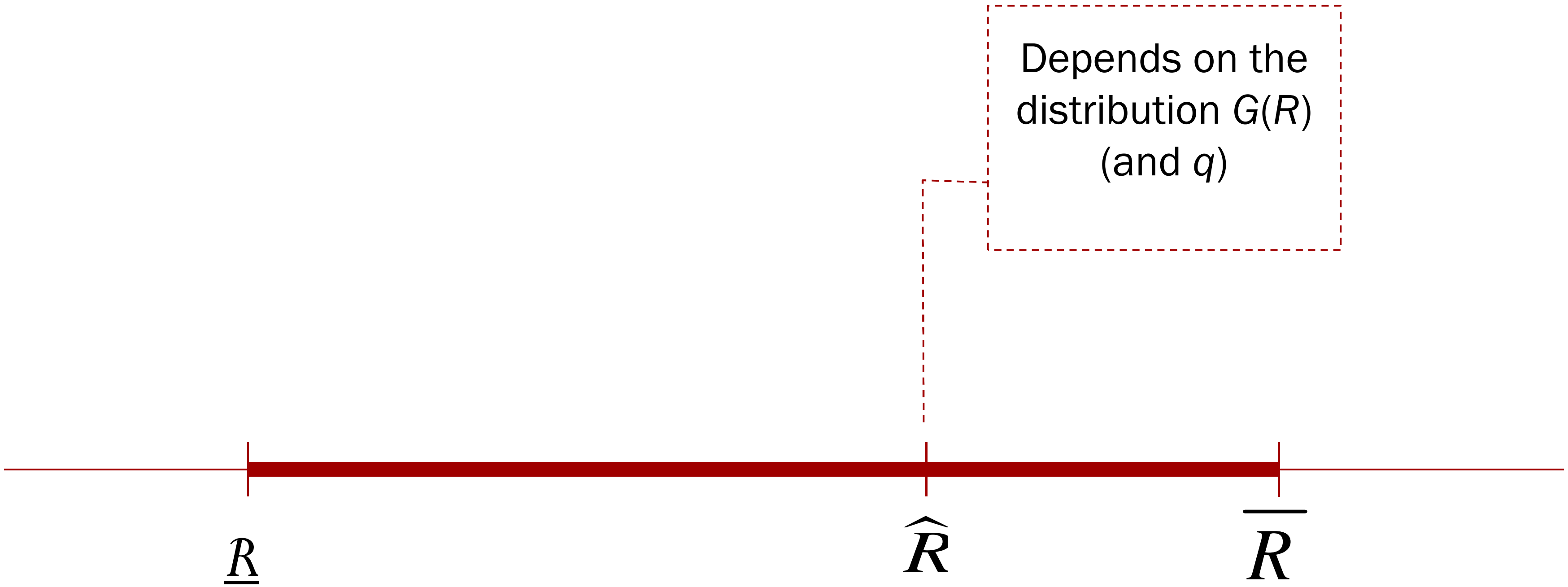
Chiappori, Macho-Stadler, Salanié, Rey (1994)

- The intuition is that the firm can always replicate in the LT contract the optimal sequence of contingent ST contracts. Moreover, LT contracts are typically superior because the firm, when it signs ST contracts, can not commit to pay the senior worker a utility level higher than his reservation utility.

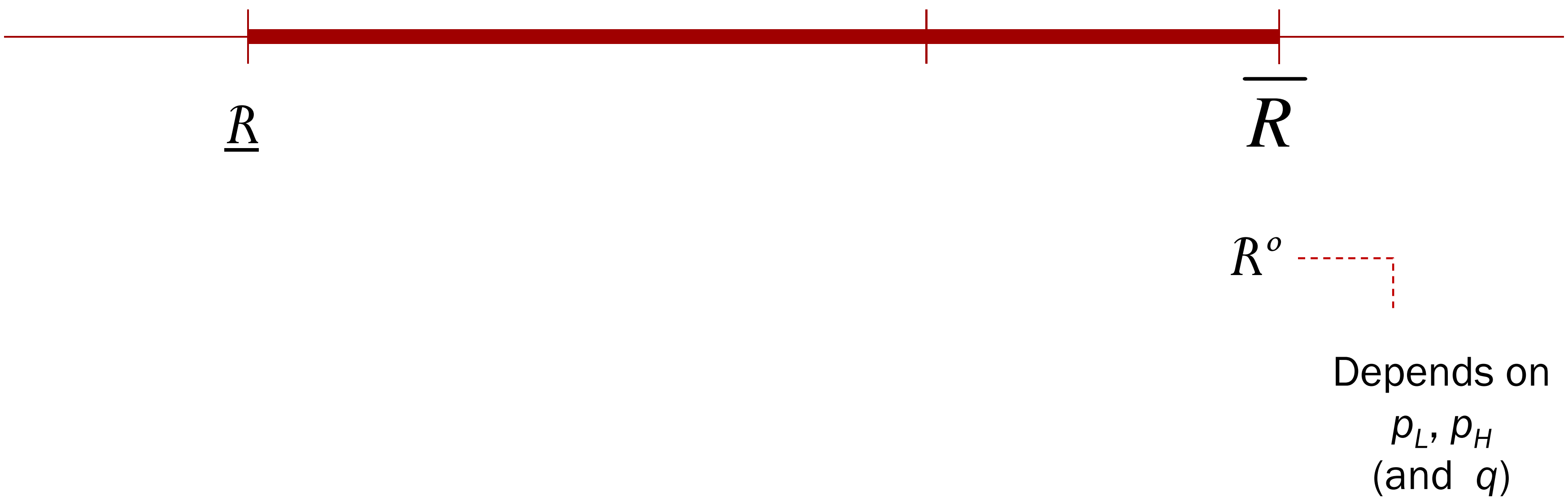
In a market

- The results depend on the distribution of principals' types and the differences of abilities of the senior workers.

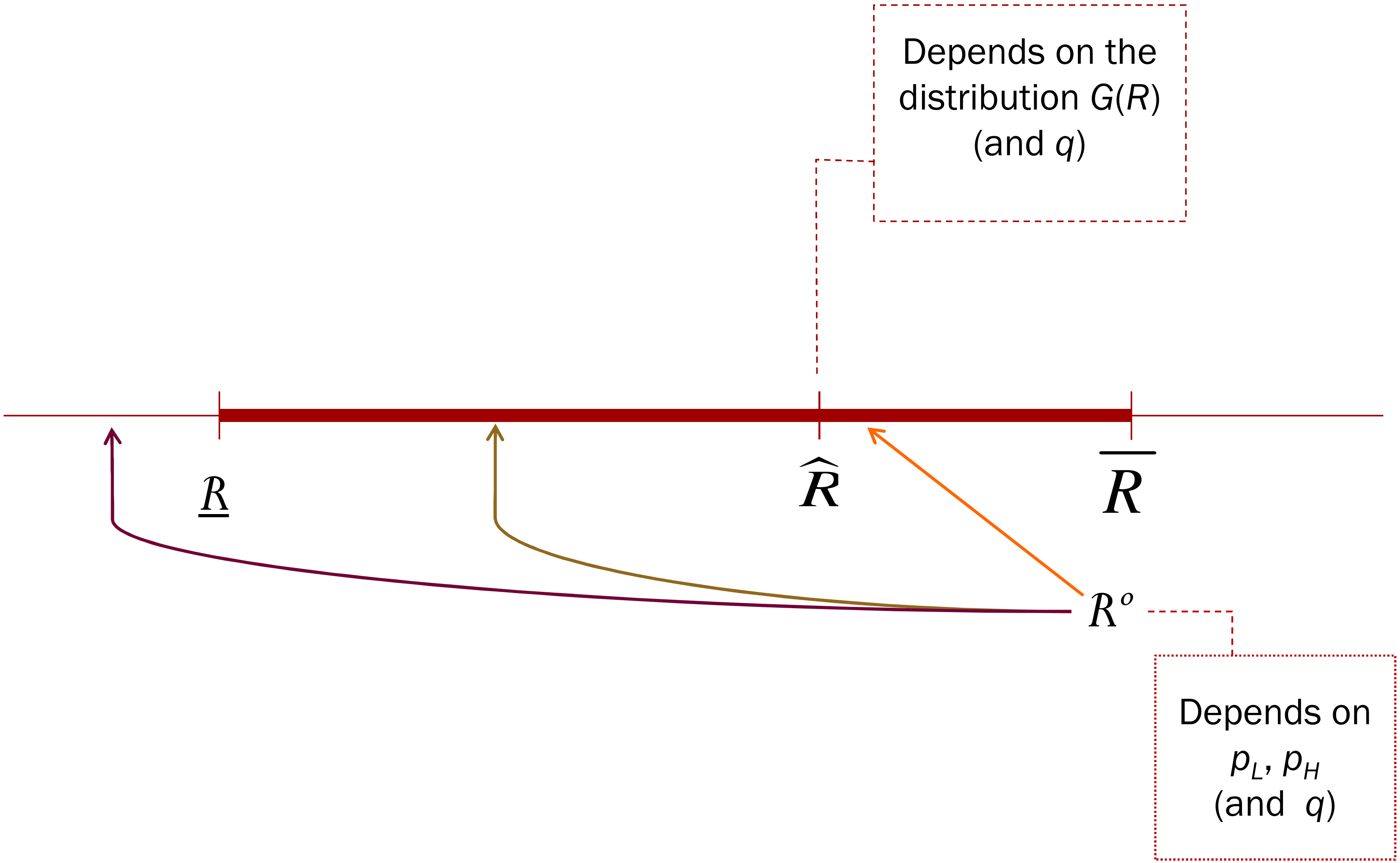
To present the result, let’s define



And also introduce R^o

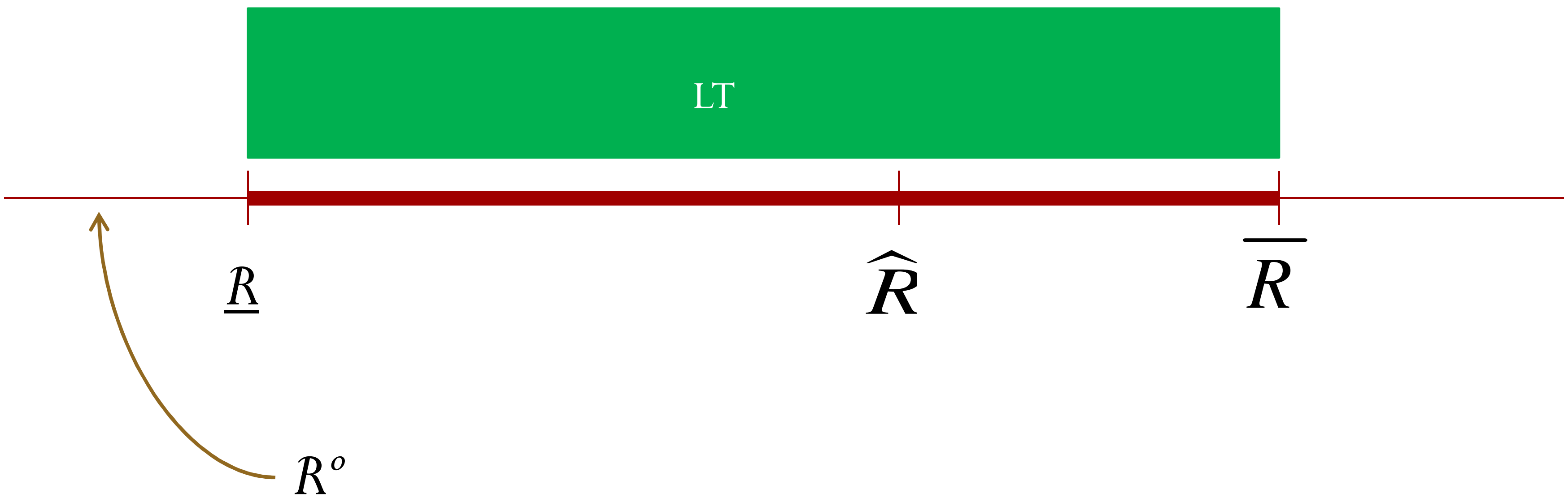


The result depends on ....

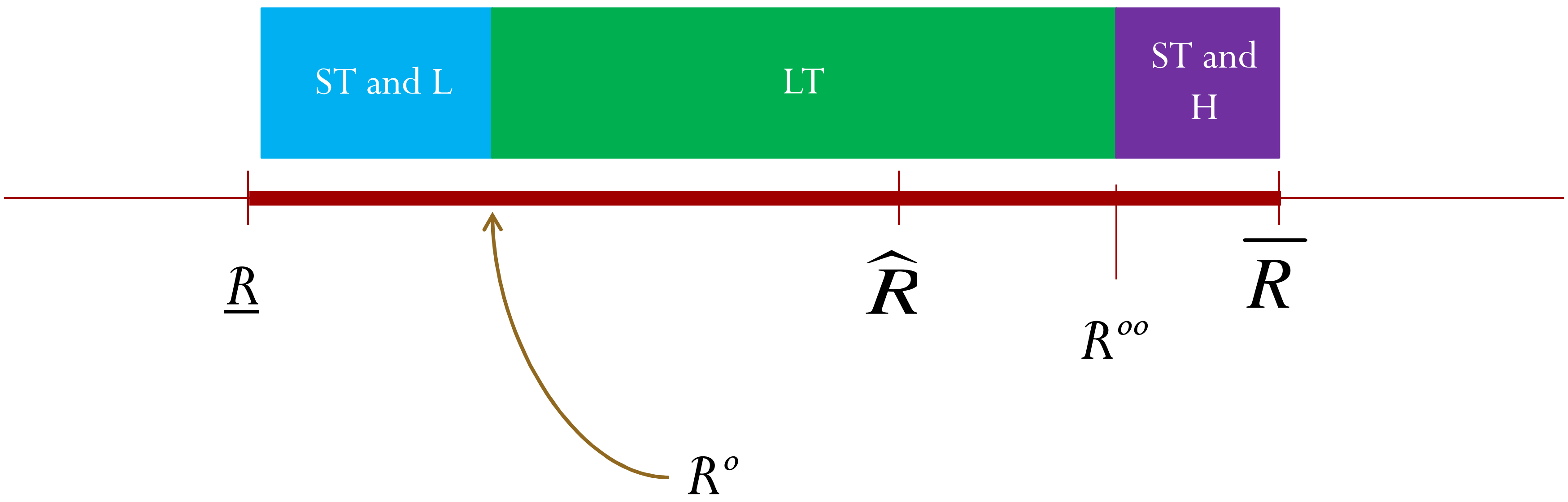


If R^o is "low"

(this equilibrium always exist and this is the prediction in the single P-A model)



If R^o is "average"



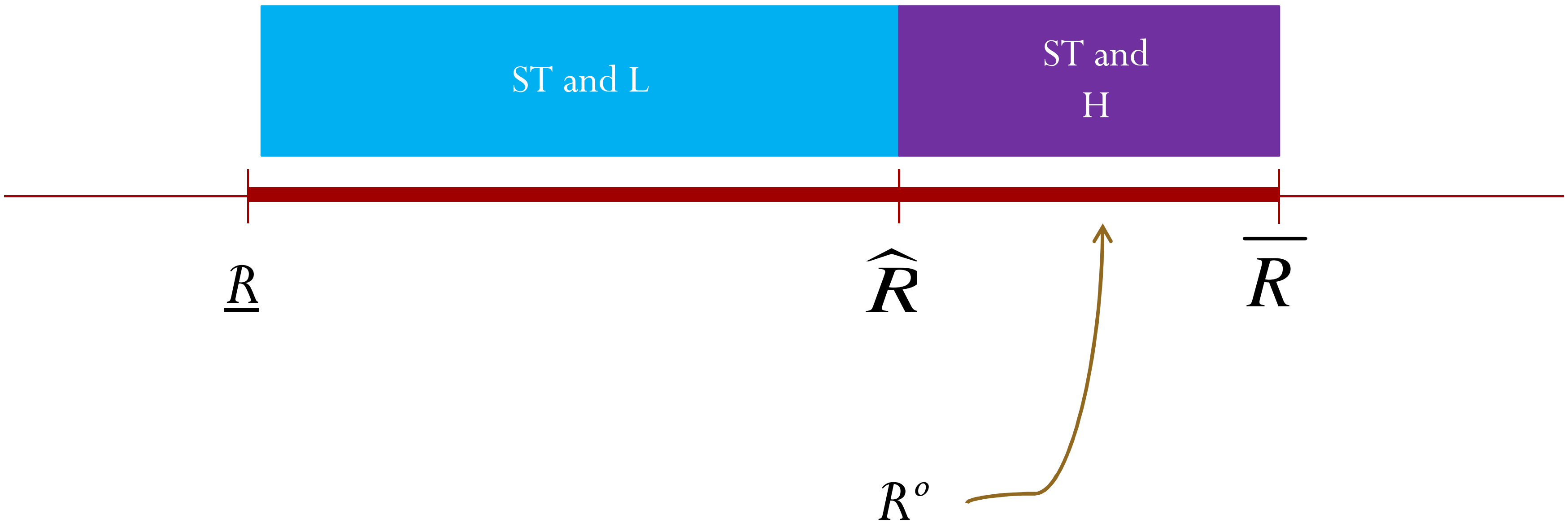
ON SURPLUS

- The "mixed" equilibrium is preferred by the firms: firms that choose ST contracts could have chosen LT contracts.

- Though all junior workers are identical when they sign their equilibrium contracts and they perform identical job, their expected utility is different depending of the type of contract:
  - Under a LT contract, a junior worker expects $(1 + \delta)\underline{U}$
  - A junior expected utility if he signs a ST contract is, at least, $\delta\,[q\, U^{oo}_{H} + (1 - q)\underline{U}]$ $(> (1 + \delta)\underline{U})$

- The equilibrium is not socially efficient (the optimal one for a planner who decides the pairs that form but cannot avoid the MH problem): the market leads to overuse of LT contracts.

If R^o is "high"



Wrapping up

- Considering the market for Principals and Agents when there is moral hazard in the relationship is a fruitful avenue of research:
  - It gives a better understanding of what is going on in the economy (e.g., relationship risk – incentives, contracts in agriculture, labor contracts, ...)
  - It gives ideas on how to conduct empirical analysis on incentives when data come from a market.

Thank you!

Kiitos!

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For more references see

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